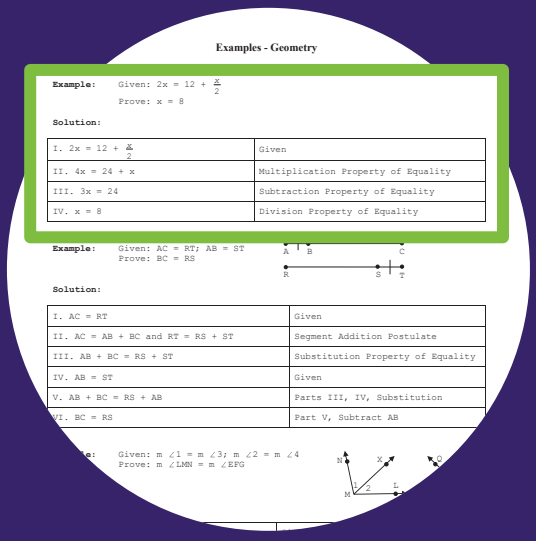
Geometry

Curriculum Sample

A Grade Ahead's rigorous, year-round math enrichment program is designed to challenge your child to a higher academic standard. Our monthly curriculum includes mathematical concepts that your child will see in school. Your child will learn and apply math concepts to real-world situations through word problems and develop strong critical thinking and analytical skills.

Each week will have an in-depth lesson (which we call Examples), homework, and answers. In these next pages, we offer a closer look at what our Examples, homework, and answers offer as well as a specific example of each.



Examples - Geometry

Example: Given: $2x = 12 + \frac{x}{2}$
Prove: $x = 8$

Solution:

I. $2x = 12 + \frac{x}{2}$	Given
II. $4x = 24 + x$	Multiplication Property of Equality
III. $3x = 24$	Subtraction Property of Equality
IV. $x = 8$	Division Property of Equality

Example: Given: AC = RT; AB = ST
Prove: BC = RS

Solution:

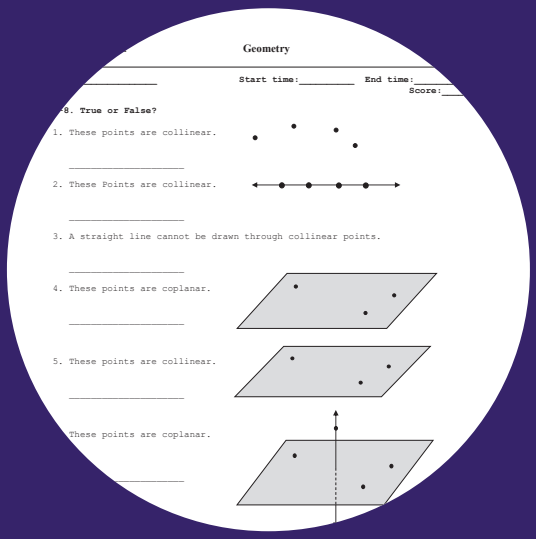
I. AC = RT	Given
II. AC = AB + BC and RT = RS + ST	Segment Addition Postulate
III. AB + BC = RS + ST	Substitution Property of Equality
IV. AB = ST	Given
V. AB + BC = RS + AB	Parts III, IV, Substitution
VI. BC = RS	Part V, Subtract AB

Given: m ∠1 = m ∠3; m ∠2 = m ∠4
Prove: m ∠LMN = m ∠EFG

Examples

To illustrate the topic, examples are provided to you and your child. These examples help demonstrate how to solve the problem or figure out the answer.

Lesson pages are titled "Examples – Geometry," answer pages are titled "Answers – Geometry," and homework pages are simply titled "Geometry."



Geometry

Start time:_____ End time:_____

Score:_____

8. True or False?

1. These points are collinear.

2. These Points are collinear.

3. A straight line cannot be drawn through collinear points.

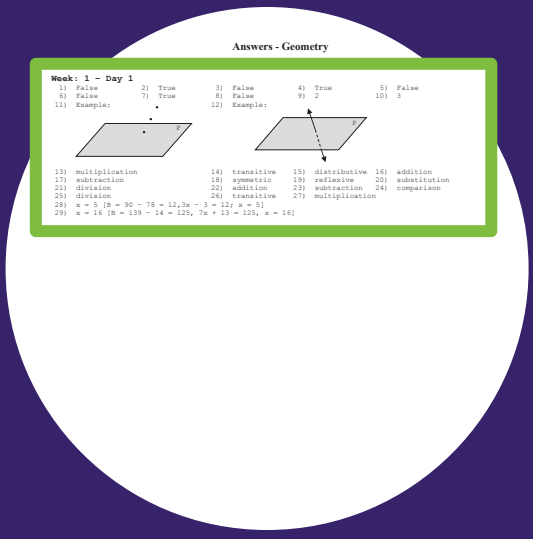
4. These points are coplanar.

5. These points are collinear.

These points are coplanar.

Homework

Each week, four days of homework are given to apply concepts from that week's lesson and reinforce the topic.



Answers - Geometry

Week: 1 – Day 1

1) False	2) True	3) False	4) True	5) False
6) False	7) True	8) False	9) 2	10) 3
11) Example:		12) Example:		

13) multiplication	14) transitive	15) distributive	16) addition
17) subtraction	18) symmetric	19) reflexive	20) substitution
21) division	22) addition	23) subtraction	24) comparison
25) division	26) transitive	27) multiplication	

28) x = 5 [8 = 90 - 78 = 12,3x - 3 = 12; x = 5]

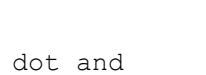
29) x = 16 [8 = 139 - 14 = 125, 7x + 13 = 125, x = 16]

Answers

Answers are provided to check your child's homework. Enter the scores into the Parent Portal to track progress and note which areas may need more work.

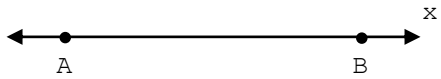
Geometry Terms, Algebraic Properties, and Inequalities

A **point** is *dimensionless* or *zero-dimensional*. A point is often represented geometrically as a dot and is named with a capital letter. (point A)



Point A

A **line** consists of infinitely-many points, extends in two directions indefinitely, and is one-dimensional. Lines are named by a single lowercase letter (line x) or after a pair of points they contain (line AB, or $\overleftrightarrow{AB}$).



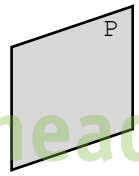
Line AB or Line x

At least two points are necessary to determine a line, since there are infinitely many lines (going in different directions) that can be drawn through any single point.

A **plane** is a flat surface that extends in two directions indefinitely. A plane has no height or thickness. Planes are specified by a capital letter (plane P) or by at least three points (plane ABCD).

At least three points are necessary to determine a plane, since there are infinitely many planes that can be drawn through any two points (or any line).

Plane ABCD
(horizontal plane)



Plane P
(vertical plane)

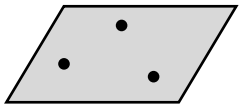
Euclidean Space is the set of all points in three dimensions.

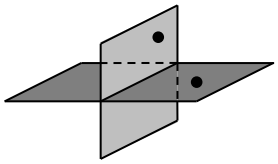
Collinear points are points that lie in the same line.


Collinear points

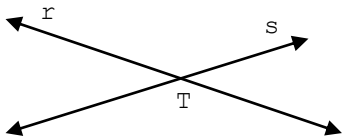

Non-collinear points

Coplanar points are points that lie in the same plane.

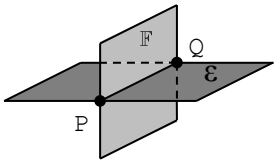

Coplanar points


Non-coplanar points

To **intersect** is to meet, cut across, or overlap.



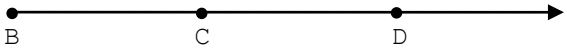
Line r and line s intersect at T



Plane ε and plane $\mathbb{F}$ intersect at $\overleftrightarrow{PQ}$

A **line segment** ($\overline{AD}$) is part of a line consisting of two points and all of the points between them. The points on the end are called **endpoints**.

A **ray** ($\overrightarrow{BD}$) is part of a line with one endpoint that extends indefinitely in the other direction. The endpoint of a ray is always named first. The arrow above the two points always points to the right, despite the direction of the figure.



Postulates or **axioms** are statements that are accepted as true without proof. Such statements are normally considered "self-evident."

Theorems are statements that have been proved by deduction from postulates or other theorems.

POSTULATE 1: ***Segment Addition Postulate***

If B is a point on the line segment $\overline{AC}$, then m $\overline{AB}$ + m $\overline{BC}$ = m $\overline{AC}$.

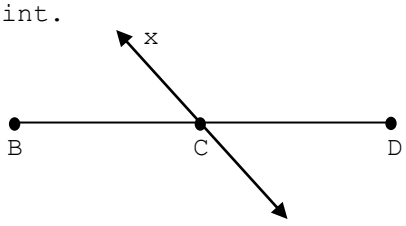
Note: m $\overline{AB}$ means the measure of $\overline{AB}$

Congruent segments are segments that are equal in length ($\overline{BC} = \overline{CD}$).

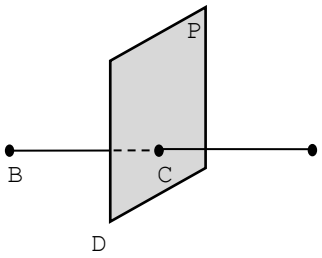
The **midpoint of a segment** divides the segment into two congruent segments.

C is the midpoint of $\overline{BD}$.

A **bisector of a segment** is a line, segment, ray, or plane that passes through its midpoint.



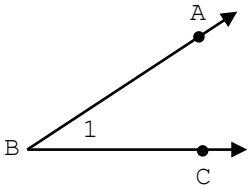
Line x is a bisector of $\overline{BD}$



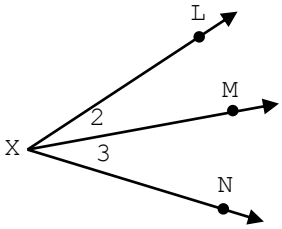
Plane P is a bisector of $\overline{BD}$

ANGLES

An **angle** (∠) consists of two rays that have the same endpoint. The endpoint is called the **vertex** of the angle. The two rays are called the **sides** of the angle.



1 angle:
∠1,
∠B, or
∠ABC



3 different angles:
∠X or ∠LXN

∠2 or ∠LXM

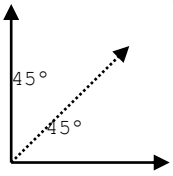
∠3 or ∠MXN

The **measure of an angle** is often abbreviated with a lowercase ***m***, as seen in Postulate 2 below.

Congruent angles are angles that are equal in measure.

Adjacent angles (∠2 and ∠3 from above) are two angles that share a vertex, share a common side, but do not have any common interior points.

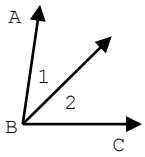
A **bisector of an angle** is a ray that divides an angle into two congruent angles.



POSTULATE 2: *Angle Addition Postulate*

If ray $\overrightarrow{XM}$ lies in the interior of ∠LXN, then m ∠LXM + m ∠MXN = m ∠LXN

Example: Solve for m ∠1 and m ∠2 if the following is true:
m ∠ABC = x + 45°; m ∠1 = x + 5; m ∠2 = x − 2



Solution: m ∠ABC = m ∠1 + m ∠2 [**Angle Addition Postulate**]
x + 45 = x + 5 + x − 2
x + 45 = 2x + 3
x = 42
m ∠1 = 42 + 5 = 47° m ∠2 = 42 − 2 = 40°

ALGEBRAIC PROPERTIES: PROPERTIES OF EQUALITY

Addition Property	If a = b, then a + c = b + c
Subtraction Property	If a = b, then a − c = b − c
Multiplication Property	If a = b, then ac = bc
Division Property	If a = b and c ≠ 0, then a/c = b/c
Substitution Property	If a = x, then a + b = x + b
Distributive Property	a(b + c) = ab + ac
Reflexive Property	a = a
Symmetric Property	If a = b, then b = a
Transitive Property	If a = b and b = c, then a = c

Example: Given: $2x = 12 + \frac{x}{2}$

Prove: $x = 8$

Solution:

I. $2x = 12 + \frac{x}{2}$	Given
II. $4x = 24 + x$	Multiplication Property of Equality
III. $3x = 24$	Subtraction Property of Equality
IV. $x = 8$	Division Property of Equality

Example: Given: AC = RT; AB = ST

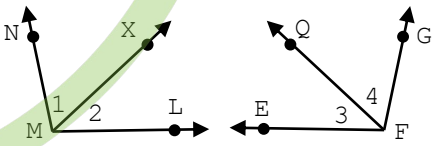
Prove: BC = RS

Solution:

I. AC = RT	Given
II. AC = AB + BC and RT = RS + ST	Segment Addition Postulate
III. AB + BC = RS + ST	Substitution Property of Equality
IV. AB = ST	Given
V. AB + BC = RS + AB	Parts III, IV, Substitution
VI. BC = RS	Part V, Subtract AB

Example: Given: m ∠1 = m ∠3; m ∠2 = m ∠4

Prove: m ∠LMN = m ∠EFG



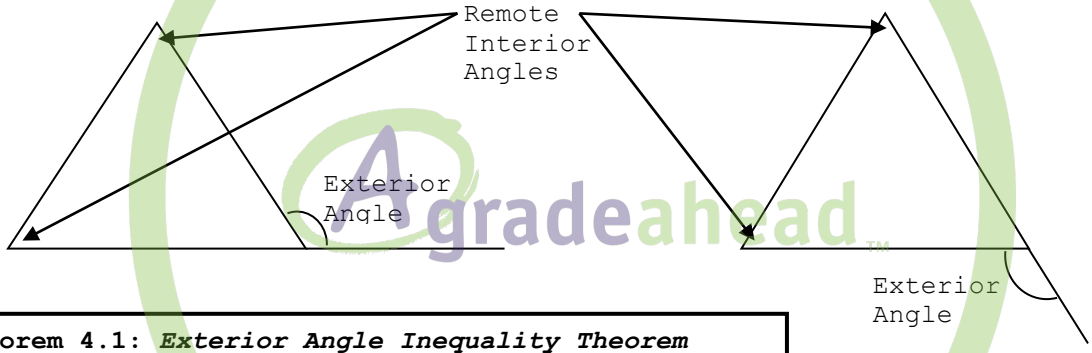
Solution:

I. m ∠1 = m ∠3; m ∠2 = m ∠4	Given
II. m ∠1 + m ∠2 = m ∠3 + m ∠4	Addition Property of Equality
III. m ∠1 + m ∠2 = m ∠LMN and m ∠3 + m ∠4 = m ∠EFG	Angle Addition Postulate
IV. m ∠LMN = m ∠EFG	Substitution Property of Equality

PROPERTIES OF INEQUALITIES:

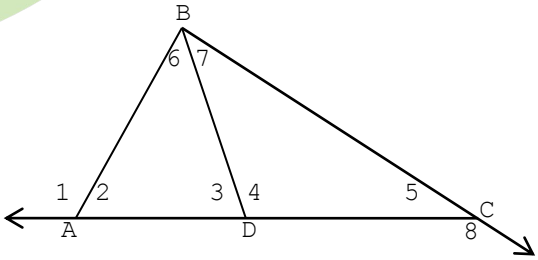
Comparison Property	For all a and b, $a > b$, $a < b$, or $a = b$
Transitive Property	If $a > b$ and $b > c$, then $a > c$
Addition Property	If $a > b$, then $a + c > b + c$
Subtraction Property	If $a > b$, then $a - c > b - c$
Multiplication Property	If $a > b$ and $c > 0$, then $ac > bc$
	If $a > b$ and $c < 0$, then $ac < bc$
Division Property	If $a > b$ and $c > 0$, then $\frac{a}{c} > \frac{b}{c}$
	If $a > b$ and $c < 0$, then $\frac{a}{c} < \frac{b}{c}$

An **exterior angle** is an angle formed by one side of a polygon and a line extended from an adjacent side. A **remote interior angle** is an angle that is not adjacent to the given exterior angle.



Theorem 4.1: ***Exterior Angle Inequality Theorem***
If an angle is an exterior angle of a triangle, then its measure is greater than the measure of either of its remote interior angles.

Example: List all of the angles that are less than m ∠1, less than m ∠4, and less then m ∠8.



Solution: ∠1 is exterior to both ΔABD and ΔABC, so it has remote interior angles ∠6, ∠3, ∠5, and the angle formed by ∠6 + ∠7. Hence m ∠6 < m ∠1, m ∠3 < m ∠1, m ∠5 < m ∠1, and m ∠7 < m ∠1.

∠4 is exterior to ΔABD only, and has remote Interior angles ∠6 and ∠2. Hence m ∠6 < m ∠4 and m ∠2 < m ∠4.

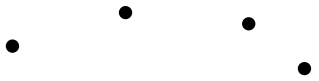
∠8 is exterior to both ΔBDC and ΔABC so it has remote interior angles ∠4, ∠7, ∠2, and the angle formed by ∠6 + ∠7. Hence m ∠4 < m ∠8, m ∠7 < m ∠8, m ∠2 < m ∠8, and m ∠6 < m ∠8.

Date:_____ **Start time:**_____ **End time:**_____

Score:____/29

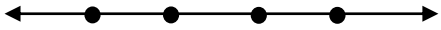
1-8. True or False?

1. These points are collinear.



_____

2. These Points are collinear.

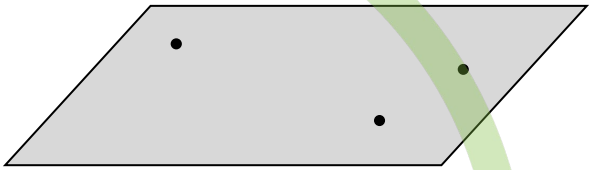


_____

3. A straight line cannot be drawn through collinear points.

_____

4. These points are coplanar.

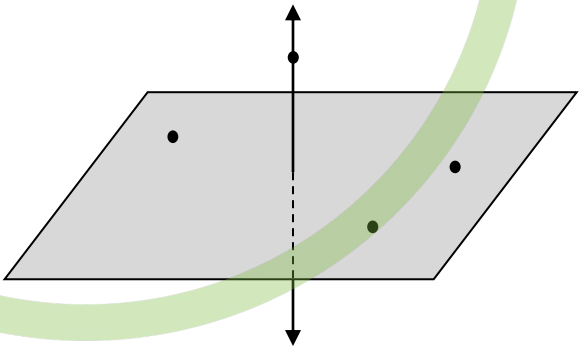


_____

5. These points are collinear.

_____

6. These points are coplanar.

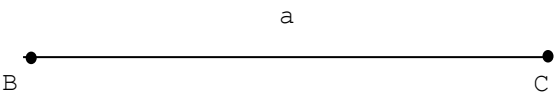


_____

7. Lines are infinitely long.

_____

8. Line a only exists between points B and C.

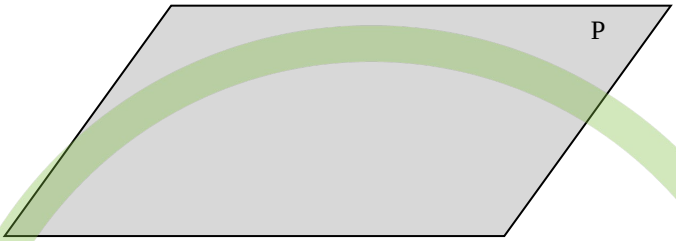


_____

9. How many points are necessary to determine a line? _____

10. How many points are necessary to determine a plane? _____

11-12. Use the diagram below to answer the following questions.



11. Draw three collinear points, with exactly one of the points on plane P.

12. Draw a line that intersects plane P at a single point.

13-21. Which algebraic property of equality corresponds to each expression?

13. If a = b, then ac = bc: _____

14. if a = b and b = c, then a = c: _____

15. a(b + c) = ab + ac: _____

16. If a = b, then a + c = b + c: _____

17. If a = b, then a – c = b – c: _____

18. If a = b, then b = a: _____

19. a = a: _____

20. If a = x, then a + b = x + b: _____

21. If a = b and c ≠ 0, then $\frac{a}{c} = \frac{b}{c}$: _____

22-27. Which property of inequality corresponds to each expression?

22. If a > b, then a + c > b + c: _____

23. If a > b, then a – c > b - c: _____

24. a > b, a < b, or a = b: _____

25. If a > b and c > 0, then $\frac{a}{c} > \frac{b}{c}$: _____

26. If a > b and b > c, then a > c: _____

27. If a > b and c < 0, then ac < bc : _____

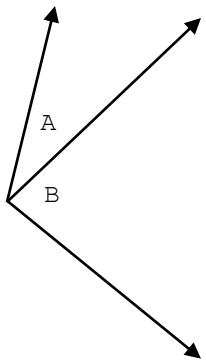
28-29. Using the figure to the right, calculate the unknown variable.

28. m ∠ A = 78; m ∠ A + m ∠ B = 90; m ∠ B = 3x - 3

x = _____

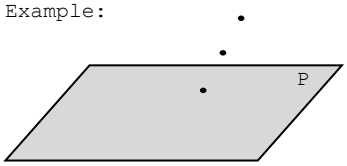
29. m ∠ A = 14; m ∠ A + m ∠ B = 139; m ∠ B = 7x + 13

x = _____

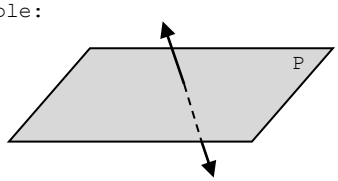


Week: 1 – Day 1

1) False 2) True 3) False 4) True 5) False
6) False 7) True 8) False 9) 2 10) 3
11) Example:



12) Example:



13) multiplication 14) transitive 15) distributive 16) addition
17) subtraction 18) symmetric 19) reflexive 20) substitution
21) division 22) addition 23) subtraction 24) comparison
25) division 26) transitive 27) multiplication
28) x = 5 [B = 90 – 78 = 12,3x – 3 = 12; x = 5]
29) x = 16 [B = 139 – 14 = 125, 7x + 13 = 125, x = 16]